

Using Computer Algebra Systems (CAS) to Build Stronger Understanding

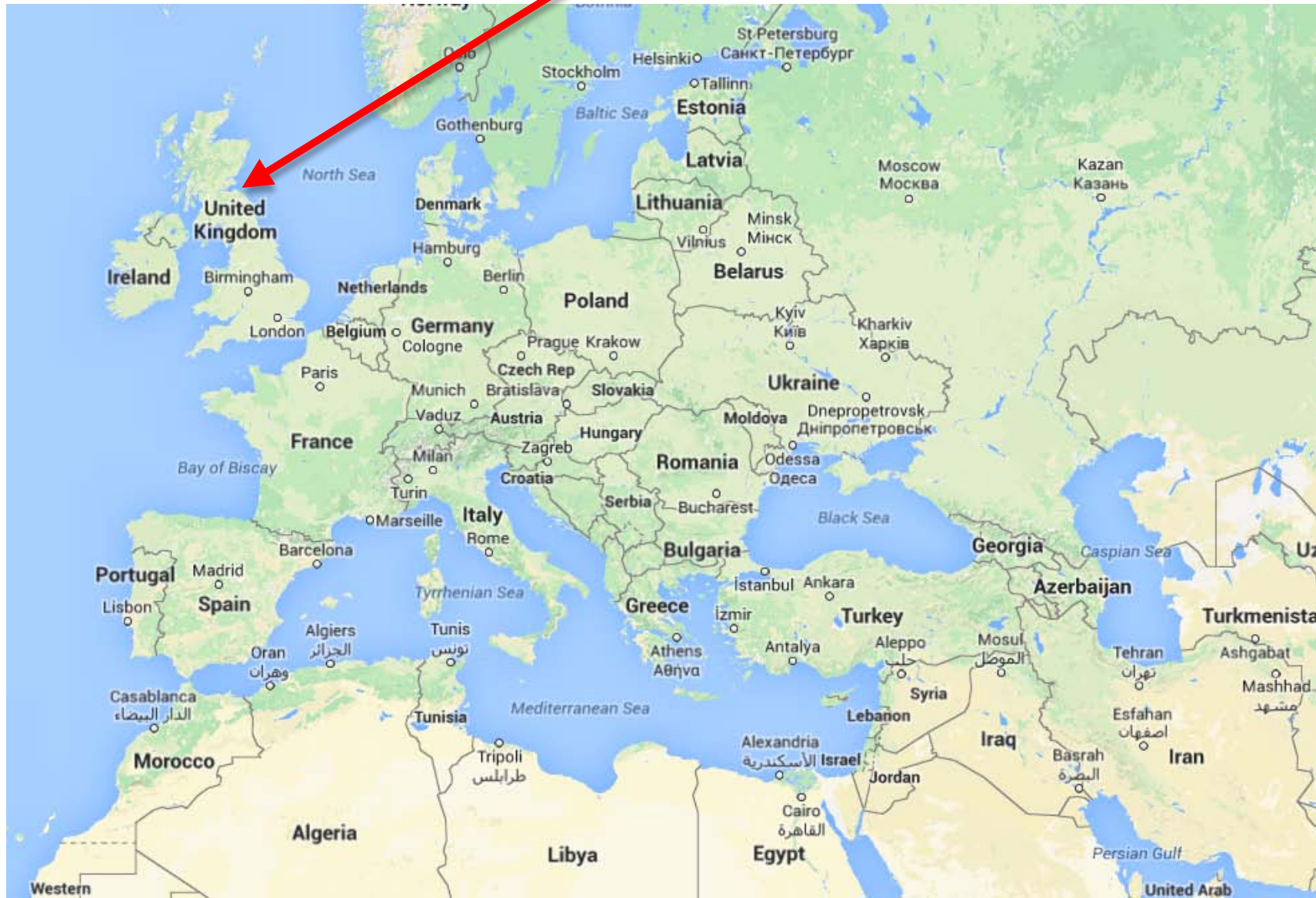


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www.calculatorsoftware.co.uk/nspire

My Home



This talk will have a....

A Beginning

Background information about me & CAS, and the remit of this talk.

A Middle

CAS activities covering various different maths topics for students aged 12-18 years.

An End

...in about 50 minutes' time!

And you can download all that you see today from



www.calculatorsoftware.co.uk/nspire

Curious Questions



CAS ... what it's not!



cas  

- cash converters**
- cash generator**
- casio**
- casualty**

Press Enter to search.



Citizens **A**dvice **S**cotland
Central **A**llocation **S**ystem
CESG **A**ssured **S**ervice
Census **A**rea **S**tatistics
Chief of **A**ir **S**taff
Cinema **A**udio **S**ociety
Computing **A**t **S**chool
Contained **A**ir **S**olutions
Centre of **A**frican **S**tudies
Consulting **A**rborist **S**ociety

Chemical **A**bstracts **S**ervice
Confirmation of **A**cceptance for **S**tudies
Cloud **A**pplications and **S**ecurity
Centre for **A**merican **S**tudies
Court of **A**rbitration for **S**port
Churches **A**gency for **S**afeguarding
Communications **A**dvisory **S**ervice
Centre for **A**tmospheric **S**cience
Circuits **A**nd **S**ystems Research Group
Community **A**ction **S**outhwark

< **G**oooooooooogle >

Previous

3 4 5 6 7 **8** 9 10 11 12

Next

Texas Instruments Nspire CX-CAS Graphic Calculator with ...

[www.amazon.co.uk](#) › ... › [Office Electronics](#) › [Calculators](#) › [Graphing](#) ▾

Texas Instruments Nspire CX-CAS Graphic Calculator with Touchpad: Amazon.co.uk: Office Products.

< **G**oooooooooogle >

Previous

4 5 6 7 8 **9** 10 11 12 13

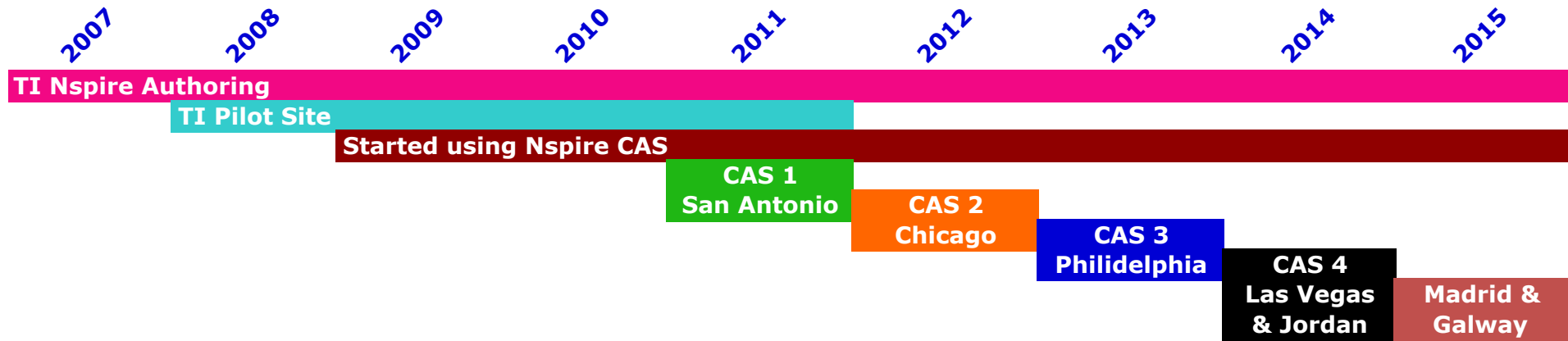
Next

CAS View - GeoGebraWiki

[wiki.geogebra.org/en/CAS_View](#) ▾

The **CAS** View allows you to use GeoGebra's **CAS** (Computer Algebra System) for symbolic computations. It consists of cells with an Input Field at the top and ...

My CAS Timeline

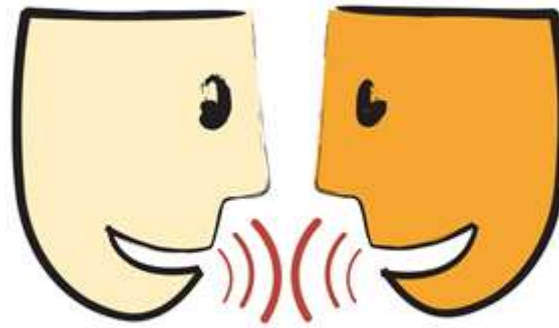


CAS Talks at TI International & European Conferences

- 2011 My first 18 months of CAS usage
- 2012 Trigonometry and Rearranging Equations
- 2013 Linear Equations and Units
- 2014 Extending CAS with functions and programs
- 2015 CAS in Statistics



Not Allowed in Exams



Allowed in Exams



Scottish Exam Arrangements Documents

“Calculators with mathematical and graphical facilities and those with **computer algebra systems (CAS)** can be utilised as powerful tools both for processing data, especially in the study of statistics, and **for reinforcing mathematical concepts.**”

“The elementary calculus studied ... is extended to differentiation of sums, products, quotients and composites of elementary functions and to integration using standard results and substitution methods respectively. ...**Computer algebra systems** can be **used extensively for consolidation and extension.**”

A Gamble? An Uphill Struggle?

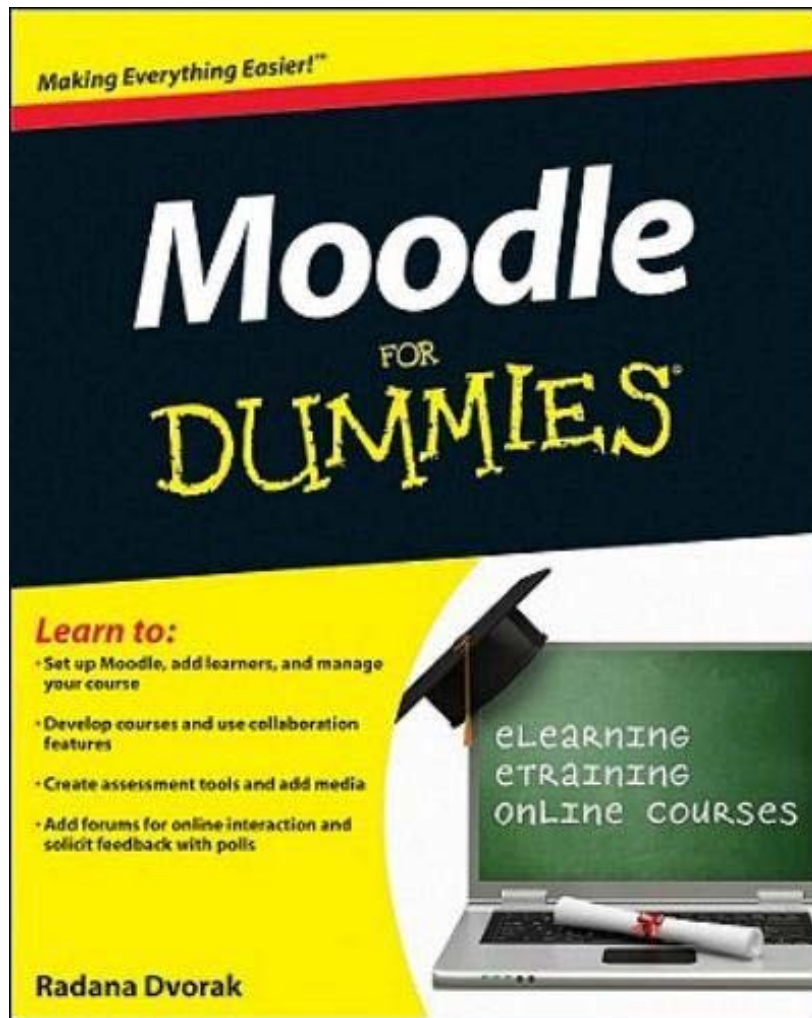


CAS Handheld of Choice

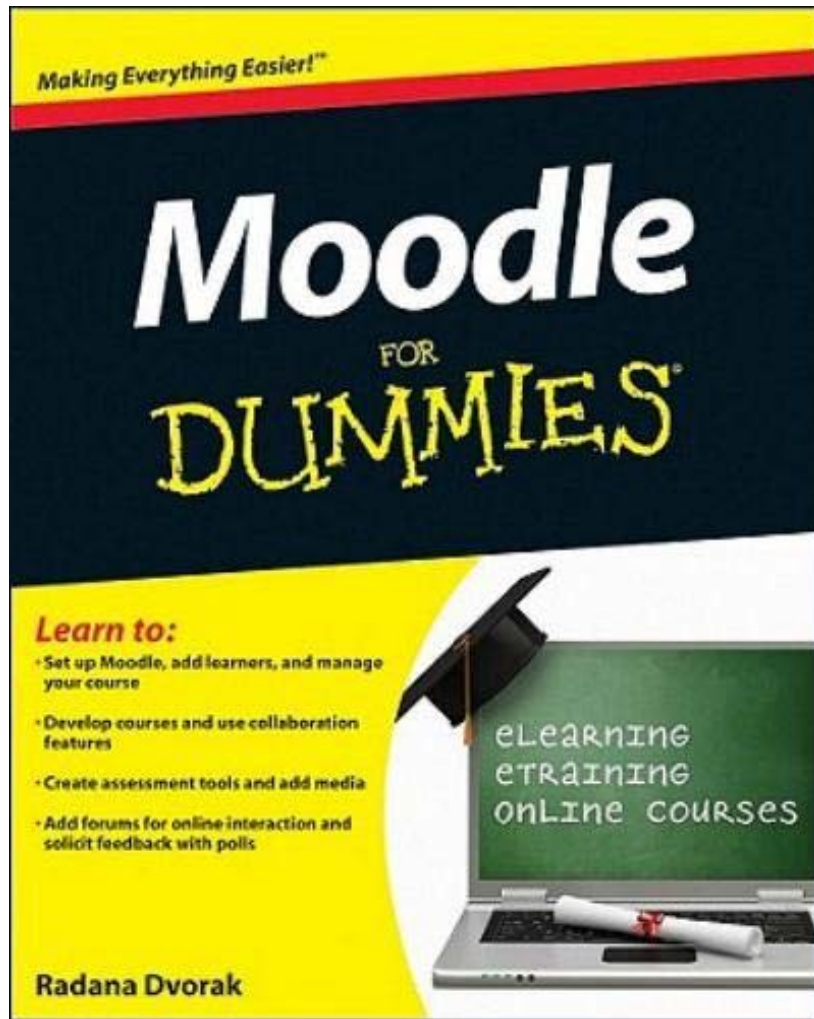


Generation Z (born 1995–2016)

... The learners of this generation are



Generation Z (born 1995–2016)



... The learners of this generation are impatient, seem to expect immediate results, and multitask with tech devices at exceptional speeds. They don't like to read instructions — most jump in and get on with it. Their expectations of technology are demanding. This generation will take to eLearning and will push boundaries.

Generation Z's Compatibility with CAS

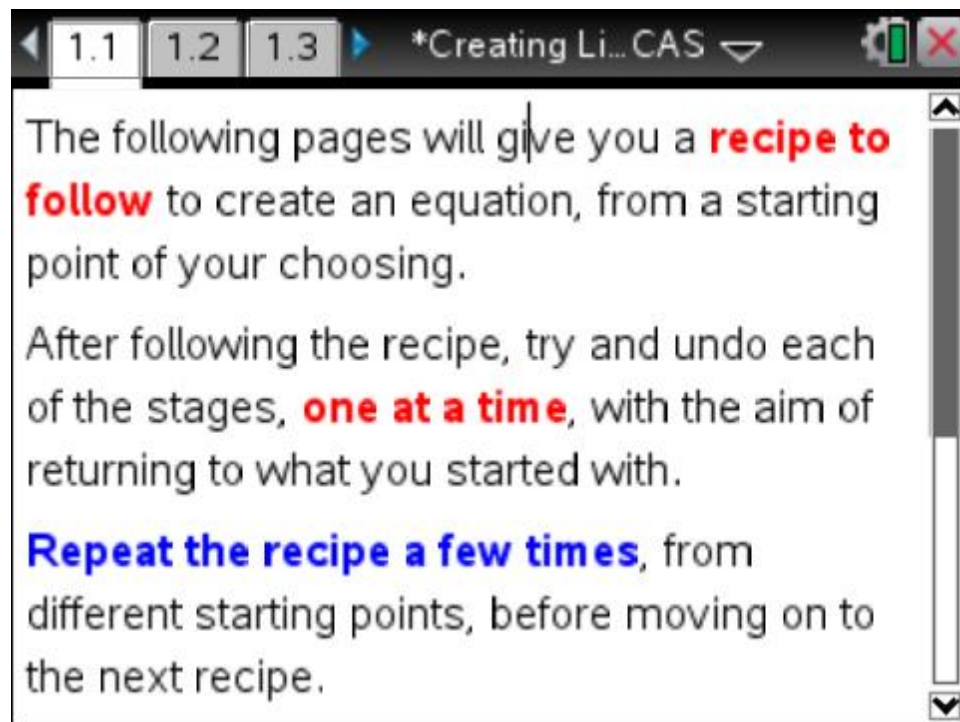
- ✓ Are play-oriented.
- ✓ Expect immediate results.
- ✓ Expect information to come to them or accessible at one click.
- ✓ Do not read instructions, especially step-by-step outlines, but jump straight in.
- ✓ Do not process as linearly as previous generations.
- ✓ Are impatient if technology is not quick enough — they find something else to do.
- ✓ Trust the medium.

Generation Z's Compatibility with CAS

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- ✓ Do not read instructions, especially step-by-step outlines, but jump straight in.
- ✓ Do not process as linearly as previous generations.
- ✓ Are impatient if technology is not quick enough — they find something else to do.
- ✓ Trust the medium.
- ✗ Look at graphics first and access text-based media last.
- ✗ Process things at "twitch speed" (ie "more than 100 images a minute.")
- ✗ Do not stay with tasks as long.
- ✗ Do not expect things to go wrong.



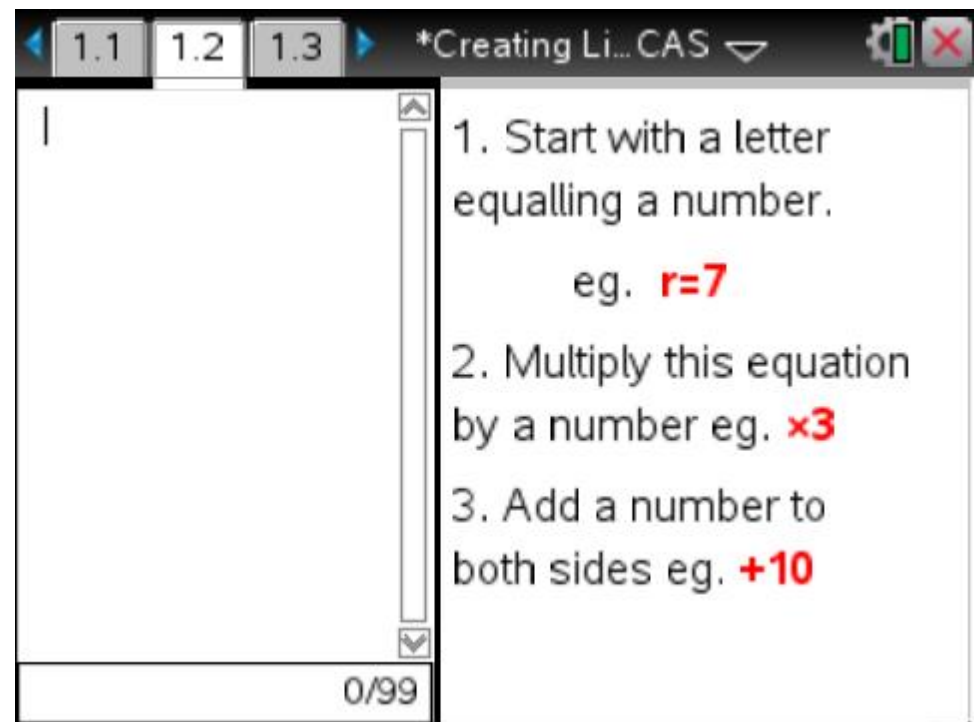
Creating Linear Equations



The following pages will give you a **recipe to follow** to create an equation, from a starting point of your choosing.

After following the recipe, try and undo each of the stages, **one at a time**, with the aim of returning to what you started with.

Repeat the recipe a few times, from different starting points, before moving on to the next recipe.



1. Start with a letter equalling a number.
eg. **r=7**

2. Multiply this equation by a number eg. **x3**

3. Add a number to both sides eg. **+10**

0/99

randomequation()

$$ax \pm b = \pm d$$

$$Ax \pm b = ax \pm d$$

$$ax \pm b = Ax \pm d$$

$$ax \pm b = cx \pm d$$

$$ax \pm b = d - cx$$

$$b - ax = cx \pm d$$

$$b - ax = d - cx$$

Simultaneous Linear Equations

The screenshot shows a window titled "Sim Equatio... CAS" with three tabs labeled "1.1", "1.2", and "1.3". The content of the window is as follows:

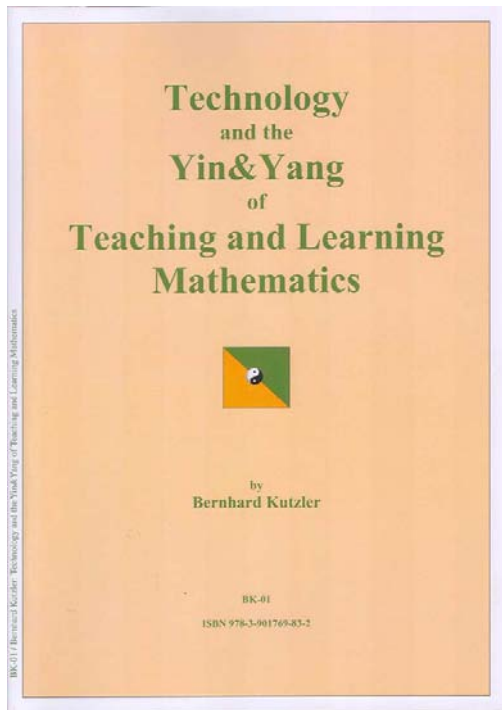
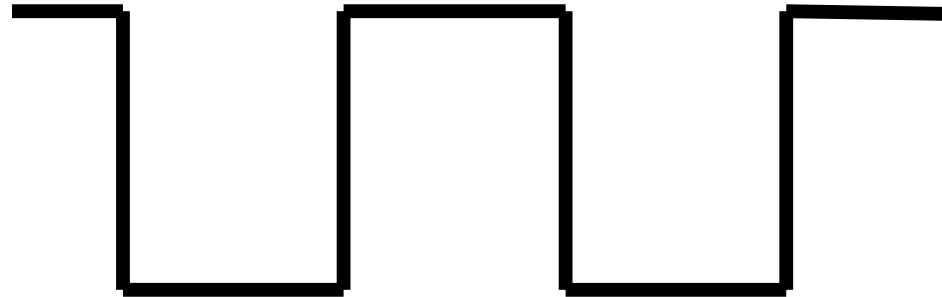
© This page shows an example of its use

$x+2 \cdot y=12$	$x+2 \cdot y=12$
$3 \cdot x-4 \cdot y=15$	$3 \cdot x-4 \cdot y=15$
<i>multiplyby2</i> ($x+2 \cdot y=12$)	$2 \cdot x+4 \cdot y=24$
<i>add</i> ($3 \cdot x-4 \cdot y=15, 2 \cdot x+4 \cdot y=24$)	$5 \cdot x=39$

© ... and then solve this as normal.

□

Avoiding Interruptions



“The picture demonstrates that a student, while trying to learn a new skill, repeatedly has to interrupt the learning process in order to perform a simplification”

Factor Pairs

The Issue

```
1.1 *Unsaved
factor(3*x^2+4*x)      x*(3*x+4)
factor(78)             2*3*13
factor(48)             2^4*3
factor(3*x^2)          3*x^2
```

More Helpful

```
1.2 1.3 1.4 *FactorPairs CAS
factorpairs(48)
5 pairs of factors
1 - 48
2 - 24
3 - 16
4 - 12
6 - 8
Done
```

Beneficial when preparing for Factorising expressions

Unexpected Behaviour – 1

The screenshot shows a software window with a title bar containing a tab labeled "1.1", the text "*Unsaved", and standard window control icons (gear, battery, and close). The main content area is a table with two columns and two rows of mathematical expressions. The first row contains $a \cdot r + a \cdot s$ in both columns. The second row contains $v \cdot b + v \cdot c$ in the left column and $(b+c) \cdot v$ in the right column. A vertical scrollbar is on the right side of the table. At the bottom right corner of the window, the text "2/99" is displayed.

$a \cdot r + a \cdot s$	$a \cdot r + a \cdot s$
$v \cdot b + v \cdot c$	$(b+c) \cdot v$

2/99

Unexpected Behaviour – 2

The image shows a presentation slide with a title bar and a table of algebraic identities. The title bar includes a tab labeled '1.1', the text '*Unsaved', and window control icons. The table contains three rows of identities, each with a horizontal line below it. The bottom right corner of the slide displays '3/99'.

$a \cdot r + a \cdot s$	$a \cdot r + a \cdot s$
$v \cdot b + v \cdot c$	$(b + c) \cdot v$
$a \cdot d + a \cdot f$	$a \cdot (d + f)$

3/99

Rearranging Equations

What do we think about when faced with rearranging these formula to make x the subject....

$$kx + m = n \qquad \frac{a}{x} + b = c \qquad r = \frac{x + p}{x - p}$$

I now know why students find rearranging so tricky!

Preparing to use CAS

4.2 4.3 4.4 ▶ *Part

$p+h$ is the same as

- $h+p$
- $-p-h$
- $-(-p-h)$
- $-(h-p)$
- $p-(-h)$

4.3 4.4 4.5 ▶ *Part 1

$a \cdot k + a \cdot p$ is the same as

- $a \cdot (p+k)$
- $a \cdot (k+p)$
- $(p+k) \cdot a$
- $(k+p) \cdot a$
- $a \cdot p + a \cdot k$
- $k \cdot a + p \cdot a$

Preparing to use CAS

◀ 4.1 4.2 4.3 ▶ Part 1 Rea

Dividing x by 2 is the same as

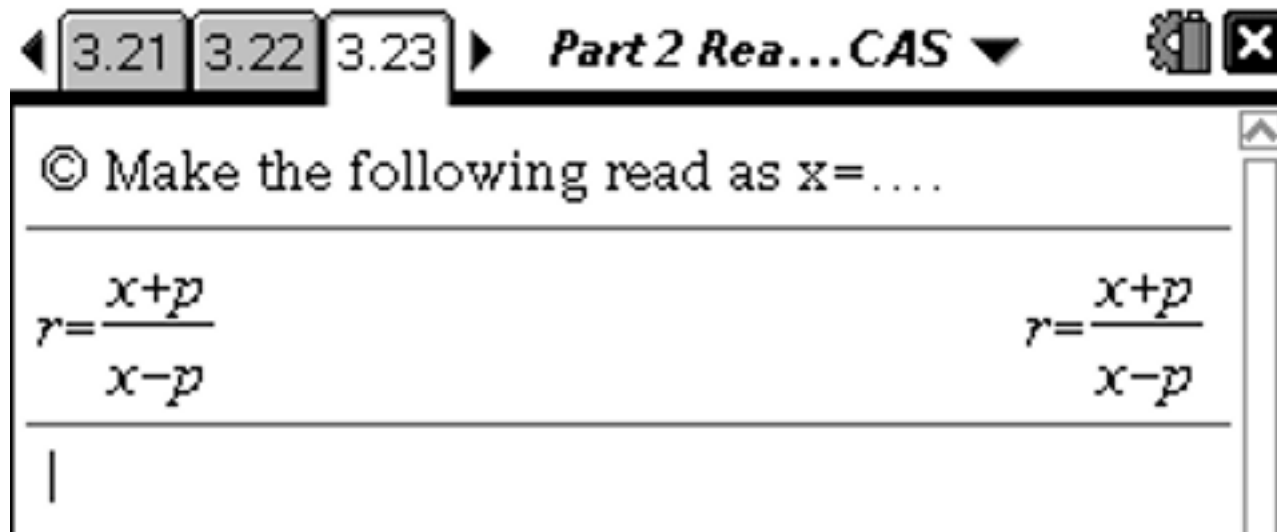
<input type="checkbox"/>	$\frac{1}{2}x$
<input type="checkbox"/>	$\frac{x}{2}$
<input type="checkbox"/>	$x \div 2$
<input type="checkbox"/>	$2x$
<input type="checkbox"/>	$x/2$
<input type="checkbox"/>	$0.5x$
<input type="checkbox"/>	$\frac{2}{x}$
<input type="checkbox"/>	\sqrt{x}

◀ 4.6 4.7 4.8 ▶ Part 1 Rea

$4a+12$ divided by 2 is

<input type="checkbox"/>	$\frac{4a}{2}+12$
<input type="checkbox"/>	$\frac{4a}{2}+\frac{12}{2}$
<input type="checkbox"/>	$4a+\frac{12}{2}$
<input type="checkbox"/>	$\frac{1}{2}(4a+12)$
<input type="checkbox"/>	$(4a+12) \div 2$
<input type="checkbox"/>	$4a+12 \div 2$

Rearranging Formulae



◀ 3.21 3.22 3.23 ▶ Part 2 Rea...CAS ▼ ⚙️ ✕

© Make the following read as $x = \dots$

$r = \frac{x+p}{x-p}$	$r = \frac{x+p}{x-p}$
-----------------------	-----------------------

|

[▶ Video of Elizabeth solving this.](#)

Elizabeth's Jotter

$$18. r = \frac{x+p}{x-p}$$

$$r(x-p) = x+p$$

$$rx-pr = x+p$$

$$rx = x+p+pr$$

$$rx - x = p + pr$$

~~$x(r-1) = p+pr$~~

$$x(r+1) = p+pr$$

$$x = \frac{p+pr}{r+1}$$

$$x = \frac{p}{r+1} + \frac{pr}{r+1}$$

$$x = p + \frac{p}{r+1}$$

$$18. r = \frac{x+p}{x-p}$$

$$r(x-p) = x+p$$

$$rx-pr = x+p$$

$$rx - pr - x = p$$

$$rx - x = p + pr$$

$$x(r-1) = p+pr$$

$$x = \frac{p+pr}{r-1} \text{ agree}$$

Elizabeth's Test Homework (2 weeks later)

$$14. \quad R = \frac{rs}{rts}$$

$$R(rts) = rs$$

$$Rrts = rs$$

$$sR = rs - RR$$

$$-rst \quad sR = -Rr$$

$$s(R-r) = -Rr$$

$$s = \frac{-Rr}{r-r}$$

] R-r?

More....

m o m e n t s

Trigonometric Equations

2.5 2.6 2.7 Solving Trig ... CAS

You will now have to use the other commands you've seen when you press the **VAR** button:

inversesine(*equation*)
inversecosine(*equation*)
inversetangent(*equation*)

2.9 2.10 2.11 *Solving Tri... CAS

© Press ENTER, then make the following read as x=....

$\tan(x)=0.247$	$\tan(x)=0.247$
$\text{inversetangent}(\tan(x)=0.247)$	$x=13.874$

3/99

Further ... Quicker

Feedback Messages

2.20 2.21 2.22 *Solving Tri... CAS

© Press ENTER, then make the following read as $x = \dots$

$5 \cdot \sin(x) - 4 = 0.$ $5 \cdot \sin(x) - 4 = 0.$

$(5 \cdot \sin(x) - 4 = 0.) + 4$ $5 \cdot \sin(x) = 4.$

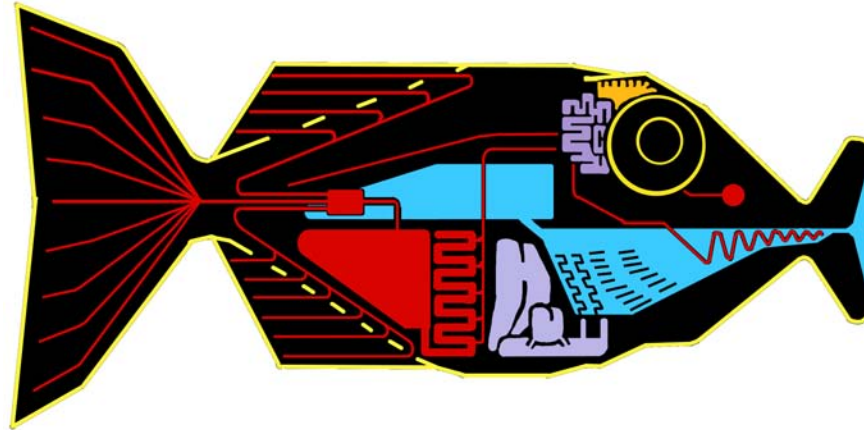
$\text{inversesine}(5 \cdot \sin(x) = 4.)$

"Inverse sine not appropriate"

4/99

Babel Fish*

**Verbal
Decisions**



**Language of
Mathematics**

* **Hitchhiker's Guide to the Galaxy, Douglas Adams, 1981**

Mathematics is....

Mathematics is....

Problem solving by reasoning

Mathematics is....

Problem solving by reasoning

Modelling

Operating

Interpreting

Reasoning

Mathematics is....

Problem solving by reasoning

Modelling

Operating

Interpreting

Reasoning

Doing → Planning

The Sine Rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

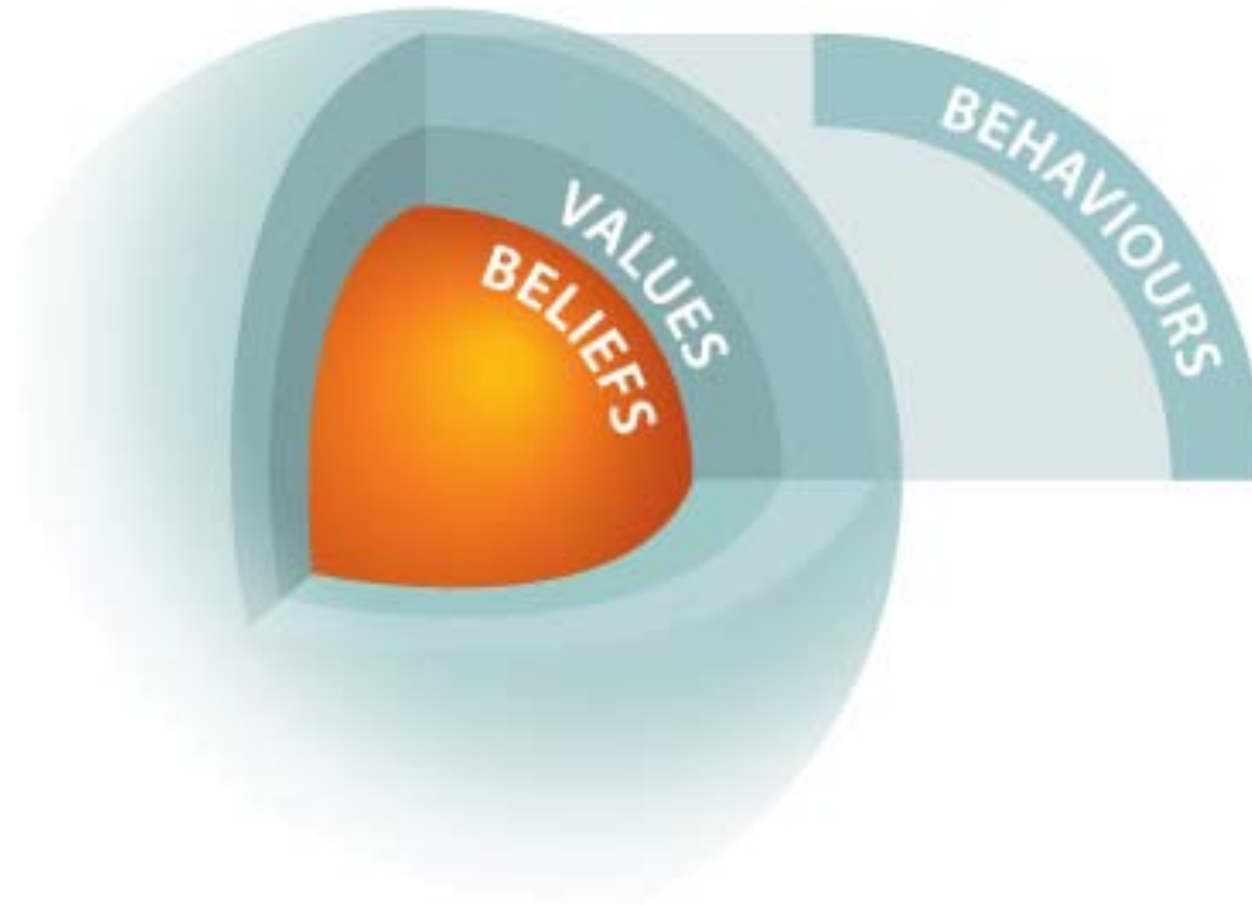


'The Lesson that Crashed'

Why do **I** use CAS?



Why would **you** use CAS?



CAS vs Established Approaches

Are traditional approaches as successful as you think?

CAS vs Established Approaches

Are traditional approaches as successful as you think?

Or are they mainly successful on specially designed tasks?

CAS vs Established Approaches

Are traditional approaches as successful as you think?

Or are they mainly successful on specially designed tasks?

**Do established methods
deserve their exalted status?**

Curious Questions?



Want Copies of Everything?
www.CalculatorSoftware.co.uk/nspire

Thank you for your attention.
Have a great Symposium!

Nevil Hopley

T³ National Trainer, Scotland & UK.

Head of Mathematics Department
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CAS User